Lattice calculation of the strangeness and electromagnetic nucleon form factors

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We report on recent lattice QCD calculations of the strangeness magnetic moment of the nucleon and the nucleon electromagnetic form factors, when we allow the electromagnetic current to connect to quark loops as well as to the valence quarks. Our result for the strangeness magnetic moment is $G_M^s(0) = -0.36 \pm 0.20$. The sea contributions from the u and d quarks are about 80% larger. However, they cancel to a large extent due to their electric charges, resulting in a smaller net sea contribution of $-0.097 \pm 0.037 \mu_N$ to the nucleon magnetic moment. As far as the neutron to proton magnetic moment ratio is concerned, this sea contribution tends to cancel out the cloud-quark effect from the Z-graphs and result in a ratio of -0.68 ± 0.04 which is close to the SU(6) relation and the experiment. The strangeness Sachs electric mean-square radius $\langle r_s^2 \rangle_E$ is found to be small and negative and the total sea contributes substantially to the neutron electric form factor.

We summarize some recent results [1] on nucleon electromagentic form factors, including the strangeness electric and magnetic form factors. The strangeness content of the nucleon has been a topic of considerable recent interest for a variety of reasons. The studies of nucleon spin structure functions in polarized deep inelastic scattering experiments at CERN and SLAC [2], combined with neutron and hyperon β decays, have turned up a surprisingly large and negative polarization from the strange quark. In addition, there is a well-known long-standing discrepancy between the pion-nucleon sigma term extracted from the low energy pion-nucleon scattering [3] and that from the octect baryon masses [4]. This discrepancy can be reconciled if a significant $\bar{s}s$ content in the nucleon [4,5] is admitted.

To address some of these issues, an experiment to measure the neutral weak magnetic form factor G_M^Z via elastic parity-violating electron scattering was recently carried out by the SAMPLE collaboration [6]. The strangeness magnetic form factor is obtained by subtracting out the nucleon magnetic form factors G_M^p and G_M^n. The reported value is $G_M^s(Q^2 = 0.1\mathrm{GeV}^2) = +0.23 \pm 0.37 \pm 0.15 \pm 0.19$. Future experiments have the promise of tightening the errors and isolating the radiative corrections so that we can hope to have a well-determined value and sign for $G_M^s(0)$.

Theoretical predictions of $G_M^s(0)$ vary widely. The values from various models and analyses range from -0.75 ± 0.30 in a QCD equalities analysis [7] to $+0.37$ in an $SU(3)$ chiral bag model [8]. While a few give positive values [8,9], most model predictions are negative with a typical range of -0.25 to -0.45. Summaries of these predictions can be found in Refs. [7,10]. A similar situation exists for the strangeness electric mean-square radius $\langle r_s^2 \rangle_E$. A number of the predictions are positive, while the others are negative. Elastic $\vec{e}\,p$ and $\vec{e}\,{}^4He$ parity-violation experiments are currently planned at TJNAF [11] to measure the asymmetry A_{LR} at forward angles to extract $\langle r_s^2 \rangle_E$. Hopefully, they will settle the issue of its sign.

In view of the large spread of theoretical predictions for both $G_M^s(0)$ and $\langle r_s^2 \rangle_E$ and in view of the fact that the experimental errors on $G_M^s(0)$ are still large, it is clearly important to perform a first-principles lattice QCD calculation in the hope that it will shed some light on these quantities.

The lattice formulation of the electromagnetic form factors has been given in detail in the past [12]. Here, we shall concentrate on the DI contribution, where the strangeness current con-

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tributes. In the Euclidean formulation, the Sachs EM form factors can be obtained by the combination of two- and three-point functions

$$G_{NN}^{\alpha\alpha}(t,\vec{p}) = \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \langle 0|\chi^{\alpha}(x)\bar{\chi}^{\alpha}(0)|0\rangle \tag{1}$$

$$G_{NV_\mu N}^{\alpha\beta}(t_f,\vec{p},t,\vec{q}) = \sum_{\vec{x}_f,\vec{x}} e^{-i\vec{p}\cdot\vec{x}_f + i\vec{q}\cdot\vec{x}} \langle 0|\chi^{\alpha}(x_f)V_\mu(x)\bar{\chi}^{\beta}(0)|0\rangle, \tag{2}$$

where χ^α is the nucleon interpolating field and $V_\mu(x)$ the vector current. With large Euclidean time separation, i.e. $t_f - t >> a$ and $t >> a$, where a is the lattice spacing,

$$\frac{\Gamma_i^{\beta\alpha} G_{NV_jN}^{\alpha\beta}(t_f,\vec{0},t,\vec{q})}{G_{NN}^{\alpha\alpha}(t_f,\vec{0})} \frac{G_{NN}^{\alpha\alpha}(t,\vec{0})}{G_{NN}^{\alpha\alpha}(t,\vec{q})} \longrightarrow \frac{\varepsilon_{ijk}q_k}{E_q+m} G_M(q^2)\ , \tag{3}$$

$$\frac{\Gamma_E^{\beta\alpha} G_{NV_4N}^{\alpha\beta}(t_f,\vec{0},t,\vec{q})}{G_{NN}^{\alpha\alpha}(t_f,\vec{0})} \frac{G_{NN}^{\alpha\alpha}(t,\vec{0})}{G_{NN}^{\alpha\alpha}(t,\vec{q})} \longrightarrow G_E(q^2) \tag{4}$$

where $\Gamma_i = \sigma_i(1+\gamma_4)/2$ and $\Gamma_E = (1+\gamma_4)/2$.

We shall use the conserved current from the Wilson action which, being point-split, yields slight variations on the above forms and these are given in Ref. [12]. Our 50 quenched gauge configurations were generated on a $16^3 \times 24$ lattice at $\beta = 6.0$. In the time direction, fixed boundary conditions were imposed on the quarks to provide larger time separations than available with periodic boundary conditions. We also averaged over the directions of equivalent lattice momenta in each configuration; this has the desirable effect of reducing error bars. Numerical details of this procedure are given in Refs. [12,13]. The dimensionless nucleon masses $M_N a$ for $\kappa = 0.154, 0.152$, and 0.148 are 0.738(16), 0.882(12), and 1.15(1) respectively. The corresponding dimensionless pion masses $m_\pi a$ are 0.376(6), 0.486(5), and 0.679(4). Extrapolating the nucleon and pion masses to the chiral limit we determine $\kappa_c = 0.1567(1)$ and $m_N a = 0.547(14)$. Using the nucleon mass to set the scale to study nucleon properties [13,14], the lattice spacing $a^{-1} = 1.72(4)$ GeV is determined. The three $\kappa' s$ then correspond to quark masses of about 120, 200, and 360 MeV respectively.

The strangeness current $\bar{s}\gamma_\mu s$ contribution appears in the DI only. In this case, we sum up the current insertion t from the nucleon source to the sink in Eqs.(3) and (4) to gain statistics [14]. The errors on the fit are obtained by jackknifing the procedure. To obtain the physical $G_M^s(q^2)$, we extrapolate the valence quarks to the chiral limit while keeping the sea quark at the strange quark mass (i.e. $\kappa_s = 0.154$). It has been shown in the chiral perturbation theory with a kaon loop that $G_M^s(0)$ is proportional to m_K, the kaon mass [15]. Thus, we extrapolate with the form $C + D\sqrt{\hat{m}+m_s}$ where $\hat{m}$ is the average u and d quark mass and m_s the strange quark mass to reflect the m_K dependence. This is the same form adopted for extracting $\langle N|\bar{s}s|N\rangle$ in Ref. [14], which also involves a kaon loop in the chiral perturbation theory.

We obtain the extrapolated $G_M^s(q^2)$ at 4 nonzero q^2 values. The errors are again obtained by jackknifing the extrapolation procedure with the covariance matrix used to include the correlation among the three valence κ's. In view of the fact that the scalar current exhibits a very soft form factor for the sea quark (i.e. $g_{S,\mathrm{dis}}(q^2)$) which has been fitted well with a monopole form [14], we shall similarly use a monopole form to extrapolate $G_M^s(q^2)$ with nonzero q^2 to $G_M^s(0)$. We find $G_M^s(0) = -0.36 \pm 0.20$. Again, the correlation among the 4 q^2 are taken into account and the error is from jackknifing the fitting procedure. This is consistent with the recent experimental value within errors (see Table 1). We also find $G_{M,\mathrm{dis}}^{u/d}(0) = -0.65 \pm 0.30$, which is about 1.8 times the size of $G_M^s(0)$. The sea contribution from the u, d, and s quarks $G_{M,\mathrm{dis}}^{u,d}(q^2)$ and $G_M^s(q^2)$ are added to the connected contributions to give the full $G_M^p(q^2)$ and $G_M^n(q^2)$.

A similar analysis is done for the strange Sachs electric form factor $G_E^s(q^2)$. We see that $G_E^s(0)$ is consistent with zero as it should be and we find that the electric mean-square radius $\langle r_s^2\rangle_E = -6dG_E^s(q^2)/dq^2|_{q^2=0} = -0.061 \pm 0.003\,\mathrm{fm}^2$.

In summary, we have calculated the s and u, d contributions to the electric and magnetic form factors of the nucleon. The individual m. m. and electric form factors from the different flavors in

Table 1
Strangeness and proton-neutron m. m. and charge radii in comparison with experiments.

	Lattice	Experiments
$G_M^s(0)$	-0.36 ± 0.20	$G_M^s(Q^2 = 0.1\mathrm{GeV}^2) = 0.23 \pm 0.37 \pm 0.15 \pm 0.19$ [6]
$G_{M,\mathrm{dis}}^u(0)$	-0.65 ± 0.30	
μ_{dis}	$-0.097 \pm 0.037\mu_N$	
μ_p	$2.62 \pm 0.07\,\mu_N$	$2.79\,\mu_N$
μ_n	$-1.81 \pm 0.07\,\mu_N$	$-1.91\,\mu_N$
μ_n/μ_p	-0.68 ± 0.04	-0.685
$\langle r_s^2 \rangle_E$	$-0.061(3)$ — $-0.16(6)\,\mathrm{fm}^2$	
$\langle r^2 \rangle_E^p$	$0.636 \pm 0.046\,\mathrm{fm}^2$	$0.659\ \mathrm{fm}^2$ [16]
$\langle r^2 \rangle_E^n$	$-0.123 \pm 0.019\,\mathrm{fm}^2$	$-0.127\,\mathrm{fm}^2$ [16]

the sea are not small, however there are large cancellations among themselves due to the electric charges of the u, d, and s quarks. We find that a negative $G_M^s(0)$ leads to a total negative sea contribution to the nucleon m. m. to make the μ_n/μ_p ratio consistent with the experiment. We also find $G_E^s(q^2)$ positive which leads to a positive total sea contribution to the neutron electric form factor $G_E^n(q^2)$. Future calculations are needed to investigate the systematic errors associated with the finite volume and lattice spacing as well as the quenched approximation.

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